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Transient response of a piezoelectric ceramic strip with an eccentric crack under electromechanical impacts

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Abstract

The transient response of a piezoelectric strip with an eccentric crack normal to the strip boundaries under applied electromechanical impacts is considered. By using the Laplace transform, the mixed initial-boundary-value problem is reduced to triple series equations, then to a singular integral equation of the first kind by introducing an auxiliary function. The Lobatto–Chebyshev collocation technique is adopted to solve numerically the resulting singular integral equation. Dynamic field intensity factors and energy release rate are obtained for both a permeable crack and an impermeable crack. The effects of the crack position and the material properties on the dynamic stress intensity factor are examined and numerical results are presented graphically.

Keywords: Piezoelectric strip; Crack; Transient response; Stress intensity factor; Energy release rate

1. Introduction

The dynamic response problem of mechanical and electrical behaviors in a piezoelectric material under various time-dependent loadings is of great significance in some practical applications such as in the detection of ultrasonic waves and has recently received much attention. In particular, many efforts in this field have been made to analyze the response features of the electric and elastic fields disturbed by cracks in a piezoelectric material subjected to dynamic electromechanical loadings. The fundamental solutions and general solutions for dynamic piezoelectricity equations for piezoelectric materials have been derived by Khutoryansky and Sosa (1995), Sosa and Khutoryansky (2001) and Ding et al. (1996), respectively. The dynamic problem of crack propagation in a piezoelectric material has been investigated in the quasi-electrostatic approximation method by Dascalu and Maugin (1995). The dynamic electroelastic behavior of a piezoelectric material has been analyzed for a semi-infinite moving crack subjected to impact loads

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by Li and Mataga (1996a,b) with the electrode boundary condition and the vacuum boundary condition at the crack surfaces, respectively. For a stationary crack of finite length in a piezoelectric material, electro-elastic field under electromechanical impacts acting on the crack surfaces has been analyzed by researchers such as Chen and Yu (1997) and Chen and Karihaloo (1999). The results are further extended to two coplanar mode-III cracks in a piezoelectric material (Chen and Worswich, 2000) and in a piezoelectric strip (Meguid and Chen, 2001). The dynamic problem of multiple mode-III cracks in a non-homogeneous material has been studied by Wang et al. (1998). The impact response problem of a mode-I crack in a piezoelectric ceramic has been considered by Shindo et al. (1999), who determined numerically the dynamic stress intensity factor and the dynamic energy release rate under the electrically permeable assumption, and by Wang and Noda (2001), who dealt with the dynamic problem of a crack in a smart laminate with two piezoelectric layers bonded to an elastic layer. For a semi-infinite crack in a piezoelectric material subjected to a concentrated electromechanical impact at the crack surfaces, a closed-form solution has been derived by Li (2001). For piezoelectric solids of finite dimension, a piezoelectric strip containing an antiplane shear crack or a mode-III crack parallel to the strip boundaries subjected to a sudden electromechanical impact has been treated by Shin et al. (2001) and Li and Fan (2002). For an antiplane shear crack normal to the strip boundaries and lying at the center of the strip, the dynamic problem has also been tackled by Chen and Meguid (2000) and Wang and Yu (2000), respectively.

This paper is concerned with the dynamic problem of a piezoelectric strip with a crack perpendicular to the strip boundaries. It is organized as follows. Section 2 gives a statement of the problem, in which the associated electric and elastic boundary conditions under the impermeable and permeable cases are given. Using the Laplace transform and the technique of variable separation, triple series equations and further a singular integral equation of the first kind are derived for both cases considered in Section 3. In Section 4, based on the Lobatto–Chebyshev collocation technique, the resulting singular integral equation is solved numerically. The dynamic stress intensity factor and the dynamic energy release rate in the physical space are obtained by a numerical inversion of the Laplace transform in Section 5. The effects of the crack position and the material properties on the normalized stress intensity factor are examined and numerical results are presented in Section 6. Finally, the conclusions are summarized.

2. Statement of the problem

Consider an infinitely long transversely isotropic piezoelectric strip of finite width h occupying the region $0 \leqslant x \leqslant h$, $-\infty < y < \infty$ with a through Griffith crack lying at $a \leqslant x \leqslant b$, $y = 0$ $(0 \leqslant a < b \leqslant h)$, as shown in Fig. 1. Here Cartesian coordinates x, y, z are the principal axes of the material symmetry while the z-axis, which is not depicted, is oriented in the poling direction of the piezoelectric strip. The crack fronts are assumed to be parallel to the z-axis and the crack surfaces are perpendicular to the strip surfaces or boundaries. When subjected to sudden applied antiplane mechanical and inplane electric impacts, the piezoelectric strip is in a state of antiplane deformation, or longitudinal shear deformation. In this case, there are only nonvanishing the out-of-plane displacement $w(x, y, t)$ and the inplane electric potential $\phi(x, y, t)$, which are independent of z, i.e.

$$u(x,y,t) = 0, \quad v(x,y,t) = 0, \quad w = w(x,y,t), \tag{1}$$

$$E_z(x,y,t) = 0, \quad E_x = E_x(x,y,t), \quad E_y = E_y(x,y,t), \tag{2}$$

where the out-of-plane displacement gives the strain components

$$\gamma_{zx} = w_{,x}, \quad \gamma_{zy} = w_{,y} \tag{3}$$

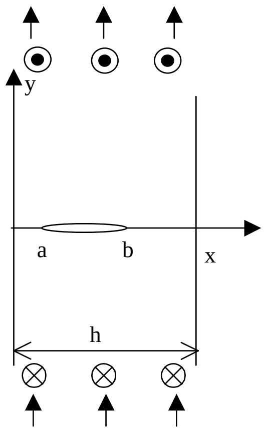

Fig. 1. A piezoelectric strip with an eccentric crack normal to the strip boundaries.

and the electric-field components are determined by

$$E_x = -\phi_{,x}, \quad E_y = -\phi_{,y}, \tag{4}$$

the comma denoting partial differentiation with respect to the suffix space variable. Based on the constitutive equations of linear piezoelectricity theory, for the present analysis stress and electric displacement are related to strain and electric field by the following equations

$$\tau_{zx} = c_{44}\gamma_{zx} - e_{15}E_x, \quad \tau_{zy} = c_{44}\gamma_{zy} - e_{15}E_y, \tag{5}$$

$$D_x = e_{15}\gamma_{zx} + \varepsilon_{11}E_x, \quad D_y = e_{15}\gamma_{zy} + \varepsilon_{11}E_y, \tag{6}$$

where c_{44}, ε_{11}, and e_{15} are the elastic stiffness measured in a constant electric field, the dielectric permittivity measured at a uniform strain, the piezoelectric constant, respectively.

Further, it follows from the equation of motion and the equilibrium equation of charges that $w(x, y, t)$ and $\phi(x, y, t)$ satisfy the basic governing differential equations for antiplane piezoelectricity dynamics, in the absence of body forces and free charges,

$$c_{44}\nabla^2 w + e_{15}\nabla^2\phi = \rho\frac{\partial^2 w}{\partial t^2}, \quad e_{15}\nabla^2 w - \varepsilon_{11}\nabla^2\phi = 0, \tag{7}$$

where ρ is the mass density of the piezoelectric ceramic strip, and ∇^2 represents the two-dimensional Laplacian operator.

The relevant mechanical and electric boundary conditions are given as follows. The strip surfaces are clearly free of stress and of electric displacement, which can be stated as

$$\tau_{zx}(0, y, t) = 0, \quad \tau_{zx}(h, y, t) = 0, \quad -\infty < y < \infty, \; t > 0, \tag{8}$$

$$D_x(0, y, t) = 0, \quad D_x(h, y, t) = 0, \quad -\infty < y < \infty, \; t > 0. \tag{9}$$

To obtain a solution of the problem, apart from boundary conditions at the strip surfaces, appropriate boundary conditions at the crack surfaces must be furnished. Of much interest from the viewpoint of fracture mechanics is the singular electroelastic field disturbed by a crack subjected to applied impact loadings. Owing to symmetry of the problem it is sufficient to consider the problem in the upper half-strip $y \geqslant 0$, so in the following we confine our attention to this region. Consequently, by superposition the problem in this region is solved under the following boundary conditions at $y = 0$,

$$\tau_{zy}(x, 0, t) = -\tau_0 f_{\mathrm{m}}(t), \quad a < x < b, \; t > 0, \tag{10}$$

$$w(x,0,t)=0, \quad 0\leqslant x\leqslant a, \;\; b\leqslant x\leqslant h, \;\; t>0, \tag{11}$$

$$\phi(x,0,t)=0, \quad 0\leqslant x\leqslant h, \;\; t>0, \tag{12}$$

for the permeable crack assumption, and

$$\tau_{zy}(x,0,t)=-\tau_0 f_{\mathrm{m}}(t), \quad a<x<b, \;\; t>0, \tag{13}$$

$$D_y(x,0,t)=-D_0 f_{\mathrm{e}}(t), \quad a<x<b, \;\; t>0, \tag{14}$$

$$w(x,0,t)=0, \quad 0\leqslant x\leqslant a, \;\; b\leqslant x\leqslant h, \;\; t>0, \tag{15}$$

$$\phi(x,0,t)=0, \quad 0\leqslant x\leqslant a, \;\; b\leqslant x\leqslant h, \;\; t>0, \tag{16}$$

for the impermeable crack assumption, where $f_{\mathrm{m}}(t)$ and $f_{\mathrm{e}}(t)$ are prescribed functions in time t, $f_{\mathrm{m}}(t)=0$ and $f_{\mathrm{e}}(t)=0$ as $t\leqslant 0$, and τ_0 and D_0 are prescribed constants.

Here, it is pointed out that the solution to the above problem is applicable to the case where two coplanar cracks ($a<|x|<b$) of equal length lying symmetrically in a piezoelectric strip ($|x|\leqslant h$). For the latter, if electromechanical loading is symmetrically applied with respect to x, we find that $w(x,y)$ and $\phi(x,y)$ are symmetrical or even with respect to x, from which $w_{,x}(x,y)$ and $\phi_{,x}(x,y)$ are odd, and so $w_{,x}(0,y)=\phi_{,x}(0,y)=0$. Furthermore, in view of the constitutive equations $\tau_{zx}(0,y)=D_x(0,y)=0$, which are equivalent to the free boundary conditions at the strip surface $x=0$. As a result, the solution to the problem stated above is also applicable to that a piezoelectric plane with a periodic array of two coplanar cracks of equal length ($a<|x-2mh|<b, \; m=0,\pm 1,\ldots,(a<b<h)$). Because of the same reason, we can conclude that $\tau_{zx}(0,y)=D_x(0,y)=0$ and $\tau_{zx}(h,y)=D_x(h,y)=0$.

3. Derivation of the singular integral equation

A simple approach for simplifying Eq. (7) is to introduce a new function

$$\varphi=\phi-\frac{e_{15}}{\varepsilon_{11}}w \tag{17}$$

and Eq. (7) then becomes

$$\nabla^2 w=\frac{1}{c_{\mathrm{s}}^2}\frac{\partial^2 w}{\partial t^2}, \quad \nabla^2\varphi=0, \tag{18}$$

where $c_{\mathrm{s}}=\sqrt{c_{\mathrm{e}}/\rho}$ denotes the shear wave velocity of a piezoelectric material, $c_{\mathrm{e}}=(c_{44}\varepsilon_{11}+e_{15}^2)/\varepsilon_{11}$ being the elastic stiffened constant identical to the one c_{44}^D measured under a constant electric displacement condition (Dieulesaint and Royer, 1980).

In order to obtain the desired electroelastic field, for convenience it is necessary to impose that the piezoelectric strip is initially at rest. Namely, the piezoelectric material is subjected to the vanishing initial conditions

$$w(x,y,0)=0, \quad \left.\frac{\partial w}{\partial t}\right|_{(x,y,0)}=0, \quad 0\leqslant x\leqslant h, \quad -\infty<y<\infty, \tag{19}$$

$$\phi(x,y,0)=0, \quad \left.\frac{\partial \phi}{\partial t}\right|_{(x,y,0)}=0, \quad 0\leqslant x\leqslant h, \quad -\infty<y<\infty. \tag{20}$$

Additionally, the solution should be sought under the regularity conditions. In other words, all the electric and elastic quantities will vanish as $y\to\infty$. Under such circumstances, using the method of variable separation (Courant and Hilbert, 1962) it is easy to verify that an appropriate solution to Eq. (18) in the Laplace transform domain may be written in a Fourier series of the form:

$$w^*(x,y,p) = \sum_{n=0}^{\infty} A_n(n,p)\exp(-\alpha y)\cos(n\beta x), \quad 0 \leqslant x \leqslant h, \ \ y \geqslant 0, \tag{21}$$

$$\varphi^*(x,y,p) = \sum_{n=0}^{\infty} B_n(n,p)\exp(-n\beta y)\cos(n\beta x), \quad 0 \leqslant x \leqslant h, \ \ y \geqslant 0, \tag{22}$$

where

$$\alpha = \sqrt{\left(\frac{n\pi}{h}\right)^2 + \left(\frac{p}{c_s}\right)^2}, \quad \beta = \frac{\pi}{h}, \tag{23}$$

$A_n(n,p)$ and $B_n(n,p)$ $(n = 0, 1, 2, \ldots)$ are unknown functions to be determined from given boundary conditions. Here the star denotes the Laplace transform of a function with respect to t, defined by

$$f^*(p) = \int_0^{\infty} f(t)\mathrm{e}^{-pt}\,\mathrm{d}t, \tag{24}$$

where p is the Laplace transform parameter.

Furthermore, it follows from (17) that electric potential $\phi(x,y,t)$ in the Laplace transform domain is given by

$$\phi^*(x,y,p) = \sum_{n=0}^{\infty}\left[\frac{e_{15}}{\varepsilon_{11}}A_n(n,p)\exp(-\alpha y) + B_n(n,p)\exp(-n\beta y)\right]\cos(n\beta x). \tag{25}$$

With the aid of constitutive equations, from (21) and (25) it is not difficult to obtain the expressions for the components of the stress, electric displacement and electric field in the Laplace transform domain in terms of $A_n(n,p)$ and $B_n(n,p)$, i.e.

$$\tau_{zx}^* = -\beta\sum_{n=1}^{\infty} n[c_e A_n \exp(-\alpha y) + e_{15}B_n\exp(-n\beta y)]\sin(n\beta x), \tag{26}$$

$$\tau_{zy}^* = -\beta\sum_{n=0}^{\infty}\left[\frac{\alpha c_e}{\beta}A_n\exp(-\alpha y) + ne_{15}B_n\exp(-n\beta y)\right]\cos(n\beta x), \tag{27}$$

$$D_x^* = \beta\varepsilon_{11}\sum_{n=1}^{\infty} nB_n\exp(-n\beta y)\sin(n\beta x), \tag{28}$$

$$D_y^* = \beta\varepsilon_{11}\sum_{n=0}^{\infty} nB_n\exp(-n\beta y)\cos(n\beta x), \tag{29}$$

$$E_x^* = \beta\sum_{n=1}^{\infty} n\left[\frac{e_{15}}{\varepsilon_{11}}A_n\exp(-\alpha y) + B_n\exp(-n\beta y)\right]\sin(n\beta x), \tag{30}$$

$$E_y^* = \beta\sum_{n=0}^{\infty}\left[\frac{\alpha e_{15}}{\beta\varepsilon_{11}}A_n\exp(-\alpha y) + nB_n\exp(-n\beta y)\right]\cos(n\beta x), \tag{31}$$

for $0 \leqslant x \leqslant h$, $y \geqslant 0$.

Here, based on the method of variable separation, a novel series expansion method is presented to solve the problem associated a piezoelectric strip with a crack. Owing to a suitable selection of the form of a solution, from expressions (26) and (28) it is readily found that boundary conditions (8) and (9) are identically fulfilled. Obviously, this method can be extended to coplanar multicracks in a strip. In what follows a crack subjected to electromechanical impacts is analyzed under the permeable and the impermeable assumptions, respectively. It is noted that for the problems involving a central crack or two coplanar cracks of equal length normal to the strip surfaces, a usual approach is to employ the Fourier transform technique to solve it (Chen and Meguid, 2000; Meguid and Chen, 2001; Wang and Yu, 2000).

3.1. The permeable case

With the help of the vanishing initial conditions (19) and (20), application of the Laplace transform to boundary conditions (10)–(12), together with the above corresponding expressions for $y = 0$, yields

$$\sum_{n=0}^{\infty}[\alpha c_e A_n + n\beta e_{15} B_n]\cos(n\beta x) = \tau_0 f_m^*(p), \quad a < x < b, \tag{32}$$

$$\sum_{n=0}^{\infty} A_n \cos(n\beta x) = 0, \quad 0 \leqslant x \leqslant a, \;\; b \leqslant x \leqslant h, \tag{33}$$

$$\sum_{n=0}^{\infty}\left[\frac{e_{15}}{\varepsilon_{11}} A_n + B_n\right]\cos(n\beta x) = 0, \quad 0 \leqslant x \leqslant h. \tag{34}$$

From (34), it is easily shown that

$$B_n = -\frac{e_{15}}{\varepsilon_{11}} A_n, \quad n = 0, 1, 2, \ldots \tag{35}$$

Eliminating B_n in (32) gives

$$\sum_{n=0}^{\infty}\left[\alpha c_e - n\beta\frac{e_{15}^2}{\varepsilon_{11}}\right] A_n \cos(n\beta x) = \tau_0 f_m^*(p), \quad a < x < b. \tag{36}$$

Then Eqs. (33) and (36) form triple series equations for A_n.

The resulting triple series equations can be further reduced to a singular integral equation by introducing a new unknown function. To achieve this, we choose A_n given by

$$A_0 = -\frac{1}{h}\int_a^b s g(s, p)\,\mathrm{d}s, \tag{37}$$

$$A_n = -\frac{2}{nh\beta}\int_a^b g(s, p)\sin(n\beta s)\,\mathrm{d}s, \quad n = 1, 2, \ldots, \tag{38}$$

where $g(s, p)$ is an auxiliary function, whose inversion of the Laplace transform denotes the derivative of the crack displacement shape or the screw dislocation density across the crack, $\partial w(x, 0, t)/\partial x$. In effect, the above may be derived by making use of the finite Fourier transform (Sneddon, 1972). Substituting (37) and (38) into (33), and recalling the known result (Gradshteyn and Ryzhik, 1980)

$$\frac{x}{2} + \sum_{n=1}^{\infty}\frac{1}{n}\sin(nx)\cos(ny) = \begin{cases} \frac{\pi}{2}, & 0 < y < x, \\ \frac{\pi}{4}, & y = x, \\ 0, & x < y < \pi, \end{cases} \tag{39}$$

we find that Eq. (33) is automatically satisfied provided that $g(x,p)$ is subjected to the constraint

$$\int_a^b g(x,p)\,\mathrm{d}x = 0, \tag{40}$$

which is apparent due to the fact that $g(x,p)$ is the Laplace transform of $\partial w(x,0,t)/\partial x$. Furthermore, by substituting (37) and (38) into (36), and employing the identity (Gradshteyn and Ryzhik, 1980)

$$\sum_{n=1}^{\infty} \sin(nx)\cos(ny) = \frac{1}{2}\frac{\sin(x)}{\cos(y)-\cos(x)}, \quad 0 < x, y < \pi, \tag{41}$$

Eq. (36) then becomes a singular integral equation of the form:

$$\frac{1}{h}\int_a^b \frac{g(s,p)\sin(\beta s)}{\cos(\beta s)-\cos(\beta x)}\,\mathrm{d}s + \frac{1}{h}\int_a^b g(s,p)T_{\mathrm{per}}(s,x,p)\,\mathrm{d}s = \frac{\tau_0}{c_{44}} f_{\mathrm{m}}^*(p), \quad a < x < b, \tag{42}$$

with

$$T_{\mathrm{per}}(s,x,p) = -\frac{1}{1-k_{\mathrm{e}}^2}\left\{\frac{ps}{c_{\mathrm{s}}} + 2\sum_{n=1}^{\infty}\left[\sqrt{1+\left(\frac{p}{\beta n c_{\mathrm{s}}}\right)^2} - 1\right]\sin(n\beta s)\cos(n\beta x)\right\}, \tag{43}$$

where $k_{\mathrm{e}} = e_{15}/\sqrt{c_{44}\varepsilon_{11} + e_{15}^2}$ denotes the electromechanical coupling coefficient.

3.2. The impermeable case

For this case, in a similar manner, by making use of the impermeable crack assumption one can derive the following equations

$$\sum_{n=0}^{\infty}[\alpha c_{\mathrm{e}} A_n + n\beta e_{15} B_n]\cos(n\beta x) = \tau_0 f_{\mathrm{m}}^*(p), \quad a < x < b, \tag{44}$$

$$\beta\varepsilon_{11}\sum_{n=0}^{\infty} nB_n\cos(n\beta x) = -D_0 f_{\mathrm{e}}^*(p), \quad a < x < b, \tag{45}$$

$$\sum_{n=0}^{\infty} A_n\cos(n\beta x) = 0, \quad 0 \leqslant x \leqslant a, \ \ b \leqslant x \leqslant h, \tag{46}$$

$$\sum_{n=0}^{\infty}\left[\frac{e_{15}}{\varepsilon_{11}} A_n + B_n\right]\cos(n\beta x) = 0, \quad 0 \leqslant x \leqslant a, \ \ b \leqslant x \leqslant h, \tag{47}$$

which are further rewritten as two decoupled systems of triple series equations for A_n and B_n,

$$\sum_{n=0}^{\infty} A_n\cos(n\beta x) = 0, \quad 0 \leqslant x \leqslant a, \ \ b \leqslant x \leqslant h, \tag{48}$$

$$\sum_{n=0}^{\infty} \alpha A_n\cos(n\beta x) = \frac{\varepsilon_{11}\tau_0 f_{\mathrm{m}}^*(p) + e_{15} D_0 f_{\mathrm{e}}^*(p)}{c_{44}\varepsilon_{11} + e_{15}^2}, \quad a < x < b, \tag{49}$$

and

$$\sum_{n=0}^{\infty} B_n \cos(n\beta x) = 0, \quad 0 \leqslant x \leqslant a, \ \ b \leqslant x \leqslant h, \tag{50}$$

$$\sum_{n=0}^{\infty} nB_n \cos(n\beta x) = -\frac{D_0}{\beta\varepsilon_{11}} f_{\rm e}^*(p), \quad a < x < b. \tag{51}$$

For the first system for A_n, still denoting A_n as (37) and (38), an analogous procedure to the above for a permeable crack results in a governing singular integral equation for $g(s,p)$ as follows:

$$\frac{1}{h}\int_a^b \frac{g(s,p)\sin(\beta s)}{\cos(\beta s)-\cos(\beta x)}\,{\rm d}s + \frac{1}{h}\int_a^b g(s,p)T_{\rm imp}(s,x,p)\,{\rm d}s = \frac{\varepsilon_{11}\tau_0 f_{\rm m}^*(p) + e_{15}D_0 f_{\rm e}^*(p)}{c_{44}\varepsilon_{11}+e_{15}^2}, \tag{52}$$

for $a < x < b$, with

$$T_{\rm imp}(s,x,p) = -\left\{\frac{ps}{c_{\rm s}} + 2\sum_{n=1}^{\infty}\left[\sqrt{1+\left(\frac{p}{\beta n c_{\rm s}}\right)^2} - 1\right]\sin(n\beta s)\cos(n\beta x)\right\}, \tag{53}$$

where $g(s,p)$ is subjected to the constraint (40).

Now, we in turn consider the second system for B_n. By representing B_n as the following integrals

$$B_0 = -\frac{1}{h}\int_a^b sq(s,p)\,{\rm d}s, \tag{54}$$

$$B_n = -\frac{2}{nh\beta}\int_a^b q(s,p)\sin(n\beta s)\,{\rm d}s, \quad n = 1,2,\ldots, \tag{55}$$

and in a similar manner, one can obtain that $q(s,p)$ satisfies

$$\frac{1}{h}\int_a^b \frac{q(s,p)\sin(\beta s)}{\cos(\beta s)-\cos(\beta x)}\,{\rm d}s = -\frac{D_0}{\varepsilon_{11}} f_{\rm e}^*(p), \quad a < x < b, \tag{56}$$

subjected to the constraint

$$\int_a^b q(s,p)\,{\rm d}s = 0. \tag{57}$$

4. Solution of the equation

It is readily seen that the singular equations derived above are similar. Due to the complicated form of the Fredholm kernels, it seems unlikely that a closed-form solution can be determined for $g(x,p)$ unless the Fredholm kernels vanish which occurs when $p \to 0$. The latter corresponds to the situation when $t \to \infty$, or the corresponding static case, which is the same in form as Eq. (56). By solving them analytically, an analytical solution in a closed form has been obtained (Li, submitted for publication). In the following, we first consider a singular integral equation of the form

$$\frac{1}{h}\int_a^b \frac{g(s,p)\sin(\beta s)}{\cos(\beta s)-\cos(\beta x)}\,{\rm d}s + \frac{1}{h}\int_a^b g(s,p)T(s,x,p){\rm d}s = P_{\rm m} f_{\rm m}^*(p) + P_{\rm e} f_{\rm e}^*(p), \quad a < x < b, \tag{58}$$

where $T(s,x,p)$ is $T_{\rm per}(s,x,p)$ for the permeable case or $T_{\rm imp}(s,x,p)$ for the impermeable case, and $P_{\rm m}$ and $P_{\rm e}$ are two relevant constants.

For the purpose of numerical computation, we introduce the following variables

$$x_1 = \cos(\beta x), \quad s_1 = \cos(\beta s), \quad a_1 = \cos(\beta a), \quad b_1 = \cos(\beta b), \tag{59}$$

and Eq. (58) is then rewritten as a standard singular integral equation with Cauchy kernel of the first kind

$$\frac{1}{\pi}\int_{b_1}^{a_1} \frac{g_1(s_1,p)}{s_1 - x_1}\,\mathrm{d}s_1 + \frac{1}{\pi}\int_{b_1}^{a_1} g_1(s_1,p)T_1(s_1,x_1,p)\,\mathrm{d}s_1 = P_\mathrm{m}f_\mathrm{m}^*(p) + P_\mathrm{e}f_\mathrm{e}^*(p), \quad b_1 < x_1 < a_1, \tag{60}$$

where

$$g_1(s_1,p) = g(s,p), \tag{61}$$

$$T_1(s_1,x_1,p) = \frac{T(s,x,p)}{\sin(\beta s)}. \tag{62}$$

Next, defining further the normalized variables $\bar{s}$ and $\bar{x}$ such that

$$s_1 = \frac{a_1 - b_1}{2}\bar{s} + \frac{a_1 + b_1}{2}, \quad x_1 = \frac{a_1 - b_1}{2}\bar{x} + \frac{a_1 + b_1}{2}, \tag{63}$$

Eq. (60), in conjunction with the constraint (40), then becomes

$$\frac{1}{\pi}\int_{-1}^{1} \frac{\bar{g}(\bar{s},p)}{\bar{s} - \bar{x}}\,\mathrm{d}\bar{s} + \frac{1}{\pi}\int_{-1}^{1} \bar{g}(\bar{s},p)\overline{T}(\bar{s},\bar{x},p)\,\mathrm{d}\bar{s} = P_\mathrm{m}f_\mathrm{m}^*(p) + P_\mathrm{e}f_\mathrm{e}^*(p), \quad -1 < \bar{x} < 1, \tag{64}$$

with the constraint

$$\int_{-1}^{1} \frac{\bar{g}(\bar{s},p)}{\sqrt{1 - \left(\frac{a_1-b_1}{2}\bar{s} + \frac{a_1+b_1}{2}\right)^2}}\,\mathrm{d}\bar{s} = 0, \tag{65}$$

where

$$\bar{g}(\bar{s},p) = g_1(s_1,p) = g(s,p), \tag{66}$$

$$\overline{T}(\bar{s},\bar{x},p) = \frac{a_1 - b_1}{2}T_1(s_1,x_1,p) = \frac{\cos(\beta a) - \cos(\beta b)}{2\sin(\beta s)}T(s,x,p). \tag{67}$$

Eq. (64) with the constraint (65) can be attacked by means of existing numerical schemes. In what follows the Lobatto–Chebyshev collocation method (Theocaris and Ioakimids, 1977; Erdogan, 1981), is utilized to determinate the numerical solution of the above equation. It is pointed out that this method has a remarkable advantage as compared to the Gauss–Chebyshev collocation method (Erdogan, 1978), since the field intensity factors at the crack tips of concern to us are obtained directly for the former, and evaluated with a complementary procedure such as extrapolation based on the determined internal values for the latter.

From the physical considerations, the function $g(x,p)$ denoting $\partial w(x,0,p)/\partial x$ in $a < x < b$ must have singularity at the crack tips, and must be integrable in $a < x < b$. Hence, $\bar{g}(\bar{x},p)$ is assumed to take the form

$$\bar{g}(\bar{x},p) = \left[P_\mathrm{m}f_\mathrm{m}^*(p) + P_\mathrm{e}f_\mathrm{e}^*(p)\right]\frac{\Omega(\bar{x},p)}{\sqrt{1-\bar{x}^2}}, \tag{68}$$

where $\Omega(\bar{x},p)$ is a bounded continuous function in the interval $|\bar{x}| \leqslant 1$, which is obtainable by using a collocation quadrature technique. Accordingly, by employing the numerical integration formula of the closed-form (Theocaris and Ioakimids, 1977; Erdogan, 1981)

$$\frac{1}{\pi}\int_{-1}^{1}\frac{1}{\bar{s}-\bar{x}_j}\frac{\Omega(\bar{s},p)}{\sqrt{1-\bar{s}^2}}\mathrm{d}\bar{s}\simeq\frac{1}{n}\sum_{i=0}^{n}\lambda_i\frac{\Omega(\bar{s}_i,p)}{\bar{s}_i-\bar{x}_j}, \tag{69}$$

where

$$\bar{x}_j=\cos\left(\frac{(2j-1)\pi}{2n}\right),\quad j=1,2,\ldots,n, \tag{70}$$

$$\bar{s}_i=\cos\left(\frac{i\pi}{n}\right),\quad i=0,1,\ldots,n, \tag{71}$$

are the zeros of Chebyshev polynomials $T_n(x)$ of the first kind of degree n and $U_{n-1}(s)$ of the second kind of degree $n-1$, respectively, and

$$\lambda_0=\lambda_n=\tfrac{1}{2},\quad \lambda_1=\cdots=\lambda_{n-1}=1. \tag{72}$$

Eq. (64) subjected to the constraint (65) is approximated by the following system of $n+1$ linear algebraic equations in $n+1$ unknown $\Omega(\bar{s}_i,p)$ $(i=0,1,\ldots,n)$:

$$\frac{1}{n}\sum_{i=0}^{n}\lambda_i\frac{\Omega(\bar{s}_i,p)}{\bar{s}_i-\bar{x}_j}+\frac{1}{n}\sum_{i=0}^{n}\lambda_i\overline{T}(\bar{s}_i,\bar{x}_j,p)\Omega(\bar{s}_i,p)=1,\quad j=1,2,\ldots,n, \tag{73}$$

$$\sum_{i=0}^{n}\frac{\lambda_i}{\sqrt{1-\left(\frac{a_1-b_1}{2}\bar{s}_i+\frac{a_1+b_1}{2}\right)^2}}\Omega(\bar{s}_i,p)=0, \tag{74}$$

the solution of which is then straightforward.

Therefore, the values of $\Omega(\bar{s},p)$ at the collocation points $\bar{s}_i$, $\Omega(\bar{s}_i,p)$, can be determined by solving the above resulting algebraic system. Especially, $\Omega(\bar{s}_0,p)$ and $\Omega(\bar{s}_n,p)$ are of particular importance, since they are exactly proportional to or related directly to the field intensity factors near the crack tips, $x=a,b$, in the Laplace transform domain. In effect, after some algebra, we find

$$\frac{1}{h}\int_a^b\frac{g(s,p)\sin(\beta s)}{\cos(\beta s)-\cos(\beta x)}\mathrm{d}s+\frac{1}{h}\int_a^b g(s,p)T(s,x,p)\,\mathrm{d}s=-[P_{\mathrm{m}}f_{\mathrm{m}}^*(p)+P_{\mathrm{e}}f_{\mathrm{e}}^*(p)]\frac{\Omega(\pm1,p)\bar{x}}{\sqrt{\bar{x}^2-1}}+\mathrm{O}(1), \tag{75}$$

for $x\simeq a-0$ or $b+0$.

Secondly, to obtain the desired crack-tip field for an impermeable crack, in addition to the solution $g(s,p)$ to Eq. (52) one must determine the solution $q(s,p)$ to Eq. (56). Owing to the particular form of Eq. (56), an analytical solution to Eq. (56) is obtainable by a similar manner to that in Li (submitted for publication), which can be expressed in the present study as

$$q(x,p)=-\frac{D_0f_{\mathrm{e}}^*(p)}{\varepsilon_{11}}L(x), \tag{76}$$

with

$$L(x)=\frac{\cos\left(\frac{\beta b}{2}\right)\left[\sec^2\left(\frac{\beta b}{2}\right)-\chi\sec^2\left(\frac{\beta x}{2}\right)\right]}{\cos\left(\frac{\beta a}{2}\right)\sqrt{\left[\tan^2\left(\frac{\beta b}{2}\right)-\tan^2\left(\frac{\beta x}{2}\right)\right]\left[\tan^2\left(\frac{\beta x}{2}\right)-\tan^2\left(\frac{\beta a}{2}\right)\right]}}, \tag{77}$$

$$\chi=\frac{\Pi(c,k)}{K(k)},\quad k=\frac{\sqrt{\tan^2\left(\frac{\beta b}{2}\right)-\tan^2\left(\frac{\beta a}{2}\right)}}{\tan\left(\frac{\beta b}{2}\right)},\quad c=\frac{\tan^2\left(\frac{\beta a}{2}\right)-\tan^2\left(\frac{\beta b}{2}\right)}{\sec^2\left(\frac{\beta b}{2}\right)}, \tag{78}$$

where K and Π denote the complete elliptical integrals of the first, and the third kinds, respectively, i.e.

$$K(k)=\int_0^{\pi/2}\frac{1}{\sqrt{1-k^2\sin^2(\theta)}}\mathrm{d}\theta,\quad \Pi(c,k)=\int_0^{\pi/2}\frac{1}{[1+c\sin^2(\theta)]\sqrt{1-k^2\sin^2(\theta)}}\mathrm{d}\theta. \tag{79}$$

5. Physical quantities of concern

In analyzing the stability of a crack in a piezoelectric material, from the viewpoint of Griffith energy balance, the energy release rate, or J-integral, is also taken as a significant fracture criterion in piezoelectric materials (Suo et al., 1992; Dascalu and Maugin, 1994; Gao et al., 1997). Prior to the presentation of the dynamic energy release rate, it is instructive to determine the distribution of the crack-tip field, which is characterized the field intensity factors.

5.1. The permeable case

For a crack under the permeable assumption, from the results obtained in the preceding section the asymptotic expression for τ_{yz}^* in the close vicinity of the crack tips in the Laplace transform domain can be expressed in terms of $\Omega(\pm 1,p)$ as follows:

$$\tau_{yz}^*(x,0,p)=\begin{cases}\dfrac{\tau_0 f_{\mathrm{m}}^*(p)\Omega(1,p)}{2}\sqrt{\dfrac{\cos(\beta a)-\cos(\beta b)}{\cos(\beta x)-\cos(\beta a)}}+\mathrm{O}(1), & x\simeq a-0,\\ -\dfrac{\tau_0 f_{\mathrm{m}}^*(p)\Omega(-1,p)}{2}\sqrt{\dfrac{\cos(\beta a)-\cos(\beta b)}{\cos(\beta b)-\cos(\beta x)}}+\mathrm{O}(1), & x\simeq b+0,\end{cases} \tag{80}$$

from which, the dynamic stress intensity factor in the Laplace transform domain can be found to be

$$K_{\mathrm{III}}^{\tau_*}(p)=\begin{cases}\tau_0 f_{\mathrm{m}}^*(p)\Omega(1,p)Y_a, & \text{for } x=a,\\ -\tau_0 f_{\mathrm{m}}^*(p)\Omega(-1,p)Y_b, & \text{for } x=b,\end{cases} \tag{81}$$

where

$$Y_a=\sqrt{\frac{h[\cos(\beta a)-\cos(\beta b)]}{2\sin(\beta a)}},\quad Y_b=\sqrt{\frac{h[\cos(\beta a)-\cos(\beta b)]}{2\sin(\beta b)}}. \tag{82}$$

Hence, by inverting the Laplace transform, the dynamic stress intensity factor in the physical space is obtained as

$$K_{\mathrm{III}}^{\tau}(t)=\tau_0 Y F_{\mathrm{m}}(t), \tag{83}$$

where Y is either Y_a or Y_b, and $F_{\mathrm{m}}(t)$ is either $F_a^m(t)$ or $F_b^m(t)$ for the crack tips $x=a$ or $x=b$, respectively. Here $F_a^m(t)$ and $F_b^m(t)$ are defined by

$$F_a^m(t)=\int_0^t f_{\mathrm{m}}(t-u)\varsigma(1,u)\,\mathrm{d}u,\quad F_b^m(t)=-\int_0^t f_{\mathrm{m}}(t-u)\varsigma(-1,u)\,\mathrm{d}u, \tag{84}$$

in which $\varsigma(\pm 1,t)$ is the inversion of the Laplace transform $\Omega(\pm 1,p)$, i.e.

$$\varsigma(\pm 1,t)=\frac{1}{2\pi\mathrm{i}}\int_{Br}\Omega(\pm 1,p)\mathrm{e}^{pt}\,\mathrm{d}p, \tag{85}$$

where Br denotes the Bromwich path of integration.

In this case, the electric field is uniform, whereas the electric displacement is singular near the crack tips, which arises from the coupling feature between electric and elastic fields. Moreover, the dependence of the intensity factors of the stress and the electric displacement is found to be

$$K_{\mathrm{III}}^{D}(t) = \frac{e_{15}}{c_{44}} K_{\mathrm{III}}^{\tau}(t). \tag{86}$$

A knowledge of the field intensity factors can be used to obtain the dynamic energy release rate in the physical space. As a result, based on the above results the dynamic energy release rate for a permeable crack for each crack tip is given by

$$G_{\mathrm{III}}(t) = \frac{1}{2c_{44}} [K_{\mathrm{III}}^{\tau}(t)]^2 = \frac{\tau_0^2}{2c_{44}} Y^2 F_{\mathrm{m}}^2(t), \tag{87}$$

where Y and $F_{\mathrm{m}}(t)$ are defined as before.

5.2. The impermeable case

In this case, with $g(s,p)$ and $q(s,p)$ in hand, asymptotic expressions for τ_{yz}^*, D_y^*, and E_y^* in the close vicinity of the crack tips are derived from (27), (29), and (31), which are omitted here. Furthermore, their intensity factors at each crack tip in the physical space can be evaluated as

$$K_{\mathrm{III}}^{\tau}(t) = \left(\tau_0 F_{\mathrm{m}}(t) + \frac{e_{15}}{\varepsilon_{11}} D_0 F_{\mathrm{e}}(t)\right) Y - \frac{e_{15}}{\varepsilon_{11}} D_0 L f_{\mathrm{e}}(t), \tag{88}$$

$$K_{\mathrm{III}}^{D}(t) = D_0 L f_{\mathrm{e}}(t), \tag{89}$$

$$K_{\mathrm{III}}^{E}(t) = -\frac{e_{15}}{\varepsilon_{11}} \frac{\varepsilon_{11}\tau_0 F_{\mathrm{m}}(t) + e_{15} D_0 F_{\mathrm{e}}(t)}{c_{44}\varepsilon_{11} + e_{15}^2} Y + \frac{1}{\varepsilon_{11}} D_0 L f_{\mathrm{e}}(t), \tag{90}$$

where L takes either L_a or L_b corresponding to the crack tip $x = a$ or $x = b$, respectively,

$$L_a = [(1 + \cos(\beta a)) - \chi(1 + \cos(\beta b))] \sqrt{\frac{2h}{\sin(\beta a)[\cos(\beta a) - \cos(\beta b)]}}, \tag{91}$$

$$L_b = (\chi - 1)[1 + \cos(\beta b)] \sqrt{\frac{2h}{\sin(\beta b)[\cos(\beta a) - \cos(\beta b)]}}, \tag{92}$$

$F_{\mathrm{m}}(t)$ and $F_{\mathrm{e}}(t)$ are defined similarly as previously.

Unlike the permeable case, the behavior of electric field and electric displacement for an impermeable crack are dominated by $r^{-1/2}$, r being the distance away from the crack tip, similar to the stress field near the crack tips. Moreover, the dynamic electric-displacement intensity factor is proportional to the prescribed electric-displacement impact function $f_{\mathrm{e}}(t)$. The plots of $K_{\mathrm{III}}^{D}(t)/D_0 f_{\mathrm{e}}(t)\sqrt{\pi l}$ or $L/\sqrt{\pi l}$, $2l = b - a$ being the crack length, against l/h for a central crack and against b/h with $a/h = 0.2$ for an eccentric crack are depicted in Fig. 2, respectively. As expected, L_a is greater than L_b when $b/h < 0.8$ or $h - b > a$, and L_b is greater than L_a when $b/h > 0.8$ or $h - b < a$. Furthermore, they are identical for a central crack in a piezoelectric strip. It is noted that electric-displacement intensity factor for a central crack and two coplanar cracks of equal length is obtained by a numerical approach in Chen and Meguid (2000) and Meguid and Chen (2001). However, in the present study the analytic solution is determined in a closed form. Further, the dynamic electric-field intensity factor has also a square-root singularity near each crack tip and a

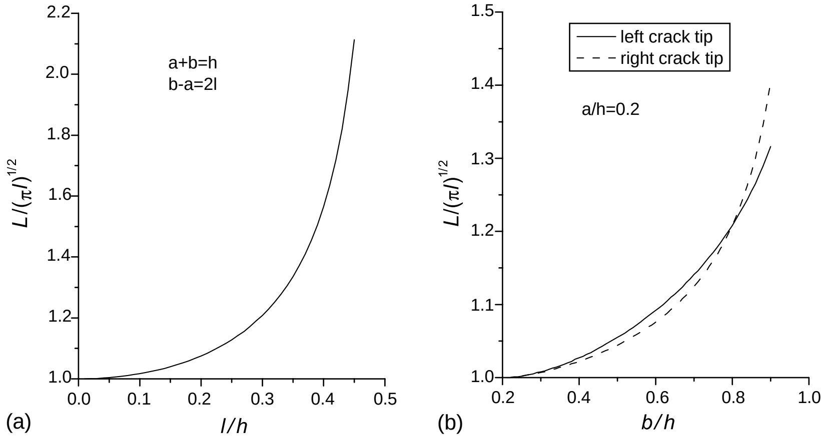

Fig. 2. The plots of L in electric displacement intensity factor (a) for a central crack against l/h, and (b) for an eccentric crack against b/h with $a/h = 0.2$.

pronounced transient feature, depending on both the mechanical and electric impacts, which is not true for a permeable crack.

Under such circumstances, the dynamic energy release rate at each crack tip for an impermeable crack is obtained as

$$G_{\mathrm{III}}(t) = \frac{1-k_{\mathrm{e}}^2}{2c_{44}}\left[\left(\tau_0 F_{\mathrm{m}}(t) + \frac{e_{15}}{\varepsilon_{11}} D_0 F_{\mathrm{e}}(t)\right)^2 Y^2 - \frac{1}{k_{\mathrm{e}}^2}\left(\frac{e_{15}}{\varepsilon_{11}} D_0\right)^2 L^2 f_{\mathrm{e}}^2(t)\right]. \tag{93}$$

6. Numerical results

In the following, the effects of the material properties and the crack geometry including the position and the length on the normalized stress intensity factor, $k^{\tau}(t) = K_{\mathrm{III}}^{\tau}(t)/K_0^{\tau}, K_0^{\tau} = \tau_0\sqrt{\pi l}$ being the corresponding static value for an infinite piezoelectric sheet containing an isolated crack of length $2l = b - a$, are examined.

To obtain dynamic stress intensity factors, instead of an analytic approach (85), a numerical inversion of the Laplace transform formulated by Stehfest (1970),

$$\varsigma(\pm 1, t) \simeq \frac{\ln(2)}{t}\sum_{n=1}^{2N} V_n \Omega\left[\pm 1, \frac{n\ln(2)}{t}\right], \tag{94}$$

with

$$V_n = (-1)^{n+N}\sum_{m=[(n+1)/2]}^{\min(n,N)} \frac{m^N (2m)!}{(N-m)!m!(m-1)!(n-m)!(2m-n)!}, \tag{95}$$

where $[(n+1)/2]$ is the integer part of the real number $(n+1)/2$, is utilized in this paper. This method has reasonable accuracy for a fairly wide range of Laplace transforms (Davies and Martin, 1979), and apart from its efficiency, it is also easy and simple. Like other numerical schemes, since inversion of the Laplace transform is an unbounded operator, the stability of the inverted Laplace transform is sensitive to the choice of N in this approach. As suggested in Stehfest (1970), N is suggested to be taken as lower integers. In the present study, based on the fact the dynamic results approach the static results as time is increased sufficiently large, the optimum N is selected as 2.

For an eccentric crack in a piezoelectric strip, it is easily found that under the permeable assumption, $k^\tau(t)$ and $G_n(t)$ depend upon the electromechanical coupling coefficient k_e, not upon electric impact acting on the crack surfaces. This is not true for an impermeable crack. For the latter, $k^\tau(t)$ and $G_n(t)$ depend upon electric impact acting on the crack surfaces, which is in stark contrast to the former.

When time-dependent function $f(t)$ is taken as $H(t)$, denoting the Heaviside unit step function, which is of much interest in practical applications, variations of $k^\tau(t)$ with the normalized time c_st/l for ZnO, PZT 65/35, and PZT-4 are illustrated in Fig. 3 for a permeable crack. The electromechanical coupling coefficients of three piezoelectric materials mentioned above are $k_e = 0.2586$, 0.4921, 0.7026, respectively (Li and Mataga, 1996a,b). For comparison, the corresponding results for a purely elastic strip, i.e. $k_e = 0$, with a crack are also depicted in Fig. 3. It is observed that the effects of the electromechanical coupling coefficient are pronounced. Similar to that for a purely elastic strip, $k^\tau(t)$ rises quite rapidly in a small time, reaching a peak, and then drops slowly to its static value for a piezoelectric strip with small electromechanical coupling coefficient. The peak will disappear with an increase of electromechanical coupling coefficient. It is seen from Fig. 3 that values of stress intensity factor near the left crack tip are identical to those near the right crack tip for a central crack (Fig. 3(a)), and are greater than those near the right crack tip when $h - b > a$ (Fig. 3(b)).

In contrast, for an impermeable crack, k_e has no any influence on $k^\tau(t)$, and the electric impact acting on the crack surfaces affect drastically $k^\tau(t)$. To examine the effects of electric impacts on stress singularity near the crack tips for an impermeable crack, in the absence of mechanical impacts, the transient response of stress field due to purely electric impacts is first considered. Curves of the variations of the dynamic stress intensity factor under various electric impacts such as $f_e(t) = H(t)$, $e^{-c_st/l}$, and $\sin(c_st/l)$ are plotted for a central crack (Fig. 4(a)) and for an eccentric crack (Fig. 4(b)). For the first two time-functions, response curve starts from a negative value and reaches a peak of response curve, while for the last time-function, response curve starts from the origin and arrives at two peaks. It is easily found that the transient feature due to sudden impact is evident at the early stage of the electric impacts, and the effects of electric impacts on stress field are slight and negligible as the normalized time lt/c_s is increased to 6. It indicates that

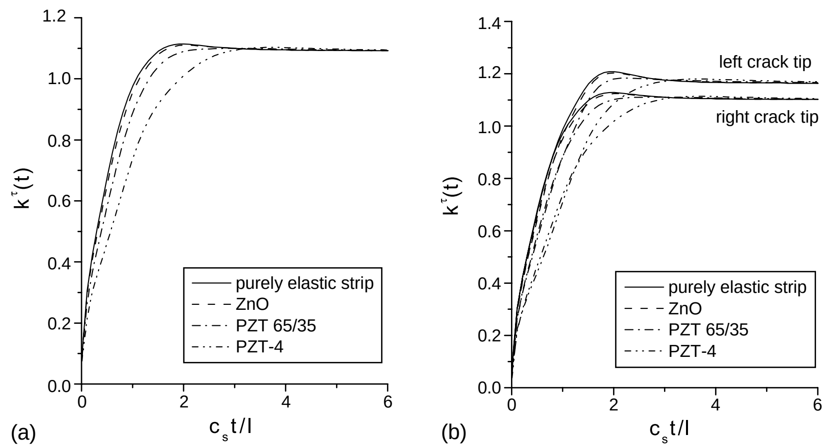

Fig. 3. Variations of the normalized stress intensity factor $k^\tau(t)$ against c_st/l for (a) $a/h = 0.2$, $b/h = 0.8$; (b) $a/h = 0.1$, $b/h = 0.5$, under the permeable assumption.

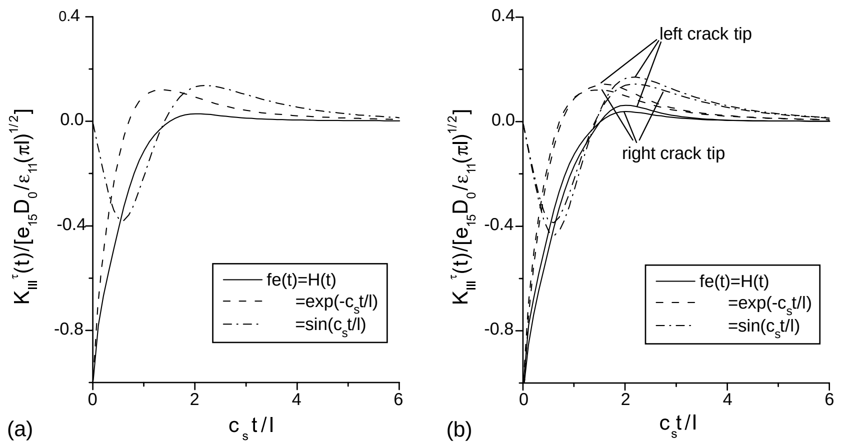

Fig. 4. Variations of stress intensity factors $K_{III}^{\tau}(t)$ due to purely various electric impacts against $c_s t/l$ for (a) $a/h = 0.2$, $b/h = 0.8$; (b) $a/h = 0.1$, $b/h = 0.5$, under the impermeable assumption.

dynamic stress field approach that for the corresponding static analysis. For the latter, electric displacement loads acting on the crack surfaces do not produce any stress field (Pak, 1990; Zhang and Tong, 1996).

To examine the dynamic effects of electric impacts on stress field in the presence of mechanical impacts, we consider a particular case. That is, assume sudden mechanical and electric impacts to be $\tau_0 H(t)$ and $D_0 H(t)$, respectively. Fig. 5 shows the variations of the normalized stress intensity factor $k^{\tau}(t)$ for a central crack and an eccentric crack. It is observed that the dynamic overshoot becomes prominent with increasing λ ($\lambda = e_{15}D_0/\varepsilon_{11}\tau_0$). A similar phenomenon can be found in Chen and Meguid (2000), Wang and Yu (2000) and Meguid and Chen (2001).

The effects of crack length on stress intensity factors are also presented graphically in Figs. 6 and 7 for a permeable and an impermeable crack, respectively. In these figures, curves of dynamic stress intensity factors near the left crack tip are plotted for different crack lengths $b/h = 0.3$, 0.6, and 0.9 and $a/h = 0.1$. From Fig. 6, the curve for a permeable crack does not to predict a peak of $k^{\tau}(t)$ when $b/h > 0.6$, and the corresponding steady-state value for $k^{\tau}(t)$ is raised with an increase of the crack length. For an impermeable

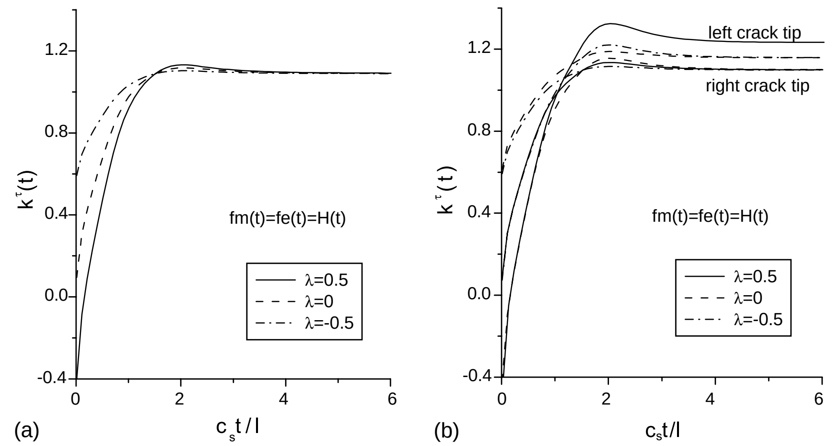

Fig. 5. Variations of the normalized stress intensity factor $k^{\tau}(t)$ against $c_s t/l$ due to mechanical and electric impacts for (a) $a/h = 0.2$, $b/h = 0.8$; (b) $a/h = 0.1$, $b/h = 0.5$, under the impermeable assumption.

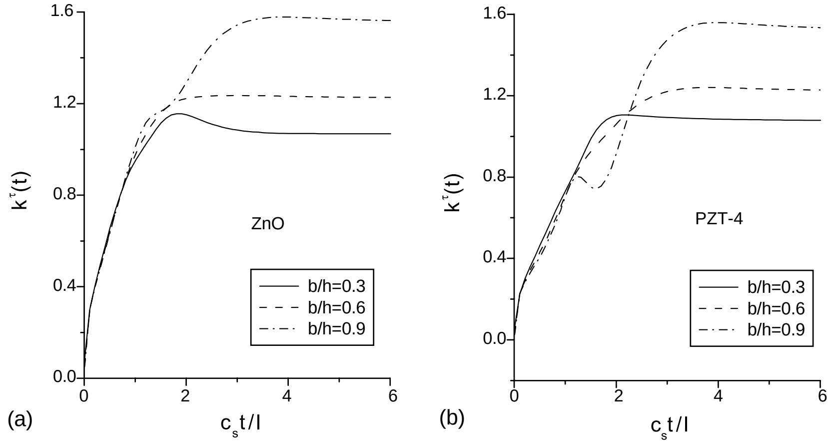

Fig. 6. Variations of the normalized stress intensity factor $k^\tau(t)$ near the left crack tip against $c_s t/l$ with different crack lengths and $a/h = 0.1$ for (a) ZnO with $k_e = 0.2586$ and (b) PZT-4 with $k_e = 0.7026$ under the permeable assumption.

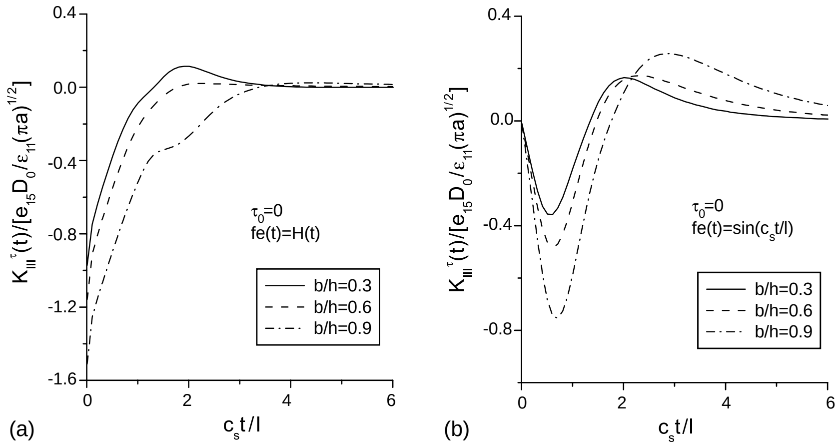

Fig. 7. Variations of the stress intensity factor $K_{\mathrm{III}}^\tau(t)$ near the left crack tip due to different electric impacts against $c_s t/l$ with different crack lengths and $a/h = 0.1$. (a) $f_e(t) = H(t)$ and (b) $f_e(t) = \sin(c_s t/l)$ under the impermeable assumption.

crack, transient responses of purely electric impacts $D_0 f_e(t)$ for $f_e(t) = H(t)$ and $f_e(t) = \sin(c_s t/l)$ are shown in Fig. 7. It is seen from Fig. 7 that negative value of stress intensity factor at the beginning stage increases and the dynamic overshoot becomes unclear with increasing the crack length when $f_e(t) = H(t)$. However, if $f_e(t) = \sin(c_s t/l)$, two peaks of curve of dynamic stress intensity factor become greater as b/h rises. Therefore, it implies that a long crack is easier to propagate than a short crack under purely electric impacts. However, this is not true for a static problem. In other words, a crack has not a tendency to propagation under purely electric displacement loads since no stress field can be induced.

7. Conclusions

The dynamic problem involving a piezoelectric strip with an eccentric crack normal to the strip boundaries is analyzed under antiplane electromechanical impacts. Using the technique of variable separation, triple series equations resulting from the mixed boundary-value problem are further converted into a singular integral equation. Dynamic stress intensity factors and energy release rate in the Laplace transform domain are obtained. By solving numerically the resulting singular integral equation and performing numerically the inverted Laplace transform, the normalized dynamic stress intensity factor is presented graphically to show the effects of the crack position and the material properties for a permeable and an impermeable crack.

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